CHAPTER 1

Basic Concepts

DIFFERENTIAL EQUATIONS

A *differential equation* is an equation involving an unknown function and its derivatives.

Example 1.1. The following are differential equations involving the unknown function y.

dependent V. = Unknown function

$\frac{dy}{dx}$

independent V.

$$\frac{dy}{dx} = 5x + 3 \tag{1.1}$$

ODE's

$$e^y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 1 \tag{1.2}$$

$$4\frac{d^3y}{dx^3} + (\sin x)\frac{d^2y}{dx^2} + 5xy = 0 \tag{1.3}$$

$$\left(\frac{d^2y}{dx^2}\right)^3 + 3y\left(\frac{dy}{dx}\right)^7 + y^3\left(\frac{dy}{dx}\right)^2 = 5x \tag{1.4}$$

$$\frac{\partial^2 y}{\partial t^2} - 4\frac{\partial^2 y}{\partial x^2} = 0 \tag{1.5}$$

→ PDE

A differential equation is an *ordinary differential equation* (ODE) if the unknown function depends on only one independent variable. If the unknown function depends on two or more independent variables, the differential equation is a *partial differential equation* (PDE). *With the exceptions of Chapters 31 and 34, the primary focus of this book will be ordinary differential equations.*

Example 1.2. Equations (*1.1*) through (*1.4*) are examples, of ordinary differential equations, since the unknown function y depends solely on the variable x. Equation (*1.5*) is a partial differential equation, since y depends on both the independent variables t and x.

The *order* of a differential equation is the order of the highest derivative appearing in the equation.

The degree " " " is the power of the highest ....

Example 1.3. Equation (*1.1*) is a first-order differential equation; (*1.2*), (*1.4*), and (*1.5*) are second-order differential equations. [Note in (*1.4*) that the order of the highest *derivative* appearing in the equation is two.] Equation (*1.3*) is a third-order differential equation.

Ex: $\frac{d^4y}{dx^4} + \left(\frac{dy}{dx}\right)^4 + y = 6$

order 4

degree 1

Prime notation $y', y'', y''', y^{(4)} \ldots y^{(n)}$
Leibniz notation $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots$
dot notation $\dot{y}, \ddot{y}, \ldots$

توضيح $y^{(4)}, y^4$

NOTATION

The expressions $y', y'', y''', y^{(4)}, \ldots, y^{(n)}$ are often used to represent, respectively, the first, second, third, fourth, ..., nth derivatives of y with respect to the independent variable under consideration. Thus, y'' represents d^2y/dx^2 if the independent variable is x, but represents d^2y/dp^2 if the independent variable is p. Observe that parentheses are used in $y^{(n)}$ to distinguish it from the nth power, y^{n}. If the independent variable is time, usually denoted by t, primes are often replaced by dots. Thus, $\dot{y}$, $\ddot{y}$, and $\dddot{y}$ represent dy/dt, d^2y/dt^2, and d^3y/dt^3, respectively.

many solutions
one solution
no solution

SOLUTIONS

A *solution* of a differential equation in the unknown function y and the independent variable x on the interval $\mathcal{I}$, is a function $y(x)$ that satisfies the differential equation identically for all x in $\mathcal{I}$.

Example 1.4. Is $y(x) = c_1 \sin 2x + c_2 \cos 2x$, where c_1 and c_2 are arbitrary constants, a solution of $y'' + 4y = 0$?

Differentiating y, we find

$$y' = 2c_1 \cos 2x - 2c_2 \sin 2x \quad \text{and} \quad y'' = -4c_1 \sin 2x - 4c_2 \cos 2x$$

Hence,

$$\begin{aligned} y'' + 4y &= (-4c_1 \sin 2x - 4c_2 \cos 2x) + 4(c_1 \sin 2x + c_2 \cos 2x) \\ &= (-4c_1 + 4c_1) \sin 2x + (-4c_2 + 4c_2) \cos 2x \\ &= 0 \end{aligned}$$

(1) infinitely sol

Thus, $y = c_1 \sin 2x + c_2 \cos 2x$ satisfies the differential equation for all values of x and is a solution on the interval $(-\infty, \infty)$.

Example 1.5. Determine whether $y = x^2 - 1$ is a solution of $(y')^4 + y^2 = -1$. (2)

Note that the left side of the differential equation must be nonnegative for every real function $y(x)$ and any x, since it is the sum of terms raised to the second and fourth powers, while the right side of the equation is negative. Since no function $y(x)$ will satisfy this equation, the given differential equation has no solution.

We see that some differential equations have infinitely many solutions (Example 1.4), whereas other differential equations have no solutions (Example 1.5). It is also possible that a differential equation has exactly one solution. Consider $(y')^4 + y^2 = 0$, which for reasons identical to those given in Example 1.5 has only one (3) solution $y \equiv 0$. → trivial solution

type of sol. A (1) *particular solution* of a differential equation is any one solution. The (2) *general solution* of a differential equation is the set of all solutions.

Example 1.6. The general solution to the differential equation in Example 1.4 can be shown to be (see Chapters 8 and 9) $y = c_1 \sin 2x + c_2 \cos 2x$. That is, every particular solution of the differential equation has this general form. A few particular solutions are: (*a*) $y = 5 \sin 2x - 3 \cos 2x$ (choose $c_1 = 5$ and $c_2 = -3$), (b) $y = \sin 2x$ (choose $c_1 = 1$ and $c_2 = 0$), and (*c*) $y \equiv 0$ (choose $c_1 = c_2 = 0$).

The general solution of a differential equation cannot always be expressed by a single formula. As an example consider the differential equation $y' + y^2 = 0$, which has two particular solutions $y = 1/x$ and $y \equiv 0$.

INITIAL-VALUE AND BOUNDARY-VALUE PROBLEMS IVP | BVP

A differential equation along with subsidiary conditions on the unknown function and its derivatives, all given at the same value of the independent variable, constitutes an *initial-value problem*. The subsidiary conditions are *initial conditions*. If the subsidiary conditions are given at more than one value of the independent variable, the problem is a *boundary-value problem* and the conditions are *boundary conditions*.

Example 1.7. The problem $y'' + 2y' = e^x$; $y(\pi) = 1$, $y'(\pi) = 2$ is an initial-value problem, because the two subsidiary conditions are both given at $x = \pi$. The problem $y'' + 2y' = e^x$; $y(0) = 1$, $y(1) = 1$ is a boundary-value problem, because the two subsidiary conditions are given at the different values $x = 0$ and $x = 1$.

A *solution* to an initial-value or boundary-value problem is a function $y(x)$ that both solves the differential equation and satisfies all given subsidiary conditions.

Solved Problems

{1.1 + 1.2 + 1.3 + 1.4 + 1.5 + 1.6 + 1.7 + 1.8 + 1.10 + 1.13}

+ degree

1.1. Determine the order, unknown function, and the independent variable in each of the following differential equations:

(*a*) $y''' - 5xy' = e^x + 1$ (*b*) $t\ddot{y} + t^2\dot{y} - (\sin t)\sqrt{y} = t^2 - t + 1$

(*c*) $s^2 \dfrac{d^2t}{ds^2} + st\dfrac{dt}{ds} = s$ (*d*) $5\left(\dfrac{d^4b}{dp^4}\right)^5 + 7\left(\dfrac{db}{dp}\right)^{10} + b^7 - b^5 = p$

(*a*) Third-order, because the highest-order derivative is the third. The unknown function is y; the independent variable is x.

(*b*) Second-order, because the highest-order derivative is the second. The unknown function is y; the independent variable is t.

(*c*) Second-order, because the highest-order derivative is the second. The unknown function is t; the independent variable is s.

(*d*) Fourth-order, because the highest-order derivative is the fourth. Raising derivatives to various powers does not alter the number of derivatives involved. The unknown function is b; the independent variable is p.

+ degree

1.2. Determine the order, unknown function, and the independent variable in each of the following differential equations:

(*a*) $y\dfrac{d^2x}{dy^2} = y^2 + 1$ (*b*) $y\left(\dfrac{dx}{dy}\right)^2 = x^2 + 1$

(*c*) $2\dddot{x} + 3\dot{x} - 5x = 0$ (*d*) $17y^{(4)} - t^6y^{(2)} - 4.2y^5 = 3\cos t$

(*a*) Second-order. The unknown function is x; the independent variable is y.

(*b*) First-order, because the highest-order derivative is the first even though it is raised to the second power. The unknown function is x; the independent variable is y.

(*c*) Third-order. The unknown function is x; the independent variable is t.

(*d*) Fourth-order. The unknown function is y; the independent variable is t. Note the difference in notation between the fourth derivative $y^{(4)}$, with parentheses, and the fifth power y^5, without parentheses.

1.3. Determine whether $y(x) = 2e^{-x} + xe^{-x}$ is a solution of $y'' + 2y' + y = 0$.

Differentiating $y(x)$, it follows that

$$y'(x) = -2e^{-x} + e^{-x} - xe^{-x} = -e^{-x} - xe^{-x}$$

$$y''(x) = e^{-x} - e^{-x} + xe^{-x} = xe^{-x}$$

Substituting these values into the differential equation, we obtain

$$y'' + 2y' + y = xe^{-x} + 2(-e^{-x} - xe^{-x}) + (2e^{-x} + xe^{-x}) = 0$$

Thus, $y(x)$ is a solution.

1.4. Is $y(x) \equiv 1$ a solution of $y'' + 2y' + y = x$?

From $y(x) \equiv 1$ it follows that $y'(x) \equiv 0$ and $y''(x) \equiv 0$. Substituting these values into the differential equation, we obtain

$$y'' + 2y' + y = 0 + 2(0) + 1 = 1 \neq x$$

Thus, $y(x) \equiv 1$ is not a solution.

1.5. Show that $y = \ln x$ is a solution of $xy'' + y' = 0$ on $\mathcal{I} = (0, \infty)$ but is not a solution on $\mathcal{I} = (-\infty, \infty)$.

On $(0, \infty)$ we have $y' = 1/x$ and $y'' = -1/x^2$. Substituting these values into the differential equation, we obtain

$$xy'' + y' = x\left(-\frac{1}{x^2}\right) + \frac{1}{x} = 0$$

Thus, $y = \ln x$ is a solution on $(0, \infty)$.

Note that $y = \ln x$ could not be a solution on $(-\infty, \infty)$, since the logarithm is undefined for negative numbers and zero.

1.6. Show that $y = 1/(x^2 - 1)$ is a solution of $y' + 2xy^2 = 0$ on $\mathcal{I} = (-1, 1)$ but not on any larger interval containing $\mathcal{I}$.

On $(-1, 1)$, $y = 1/(x^2 - 1)$ and its derivative $y' = -2x/(x^2 - 1)^2$ are well-defined functions. Substituting these values into the differential equation, we have

$$y' + 2xy^2 = -\frac{2x}{(x^2 - 1)^2} + 2x\left[\frac{1}{x^2 - 1}\right]^2 = 0$$

Thus, $y = 1/(x^2 - 1)$ is a solution on $\mathcal{I} = (-1, 1)$.

Note, however, that $1/(x^2 - 1)$ is not defined at $x = \pm 1$ and therefore could not be a solution on any interval containing either of these two points.

1.7. Determine whether any of the functions (*a*) $y_1 = \sin 2x$, (*b*) $y_2(x) = x$, or (*c*) $y_3(x) = \frac{1}{2}\sin 2x$ is a solution to the initial-value problem $y'' + 4y = 0$; $y(0) = 0$, $y'(0) = 1$.

(*a*) $y_1(x)$ is a solution to the differential equation and satisfies the first initial condition $y(0) = 0$. However, $y_1(x)$ does not satisfy the second initial condition $(y_1'(x) = 2\cos 2x; y_1'(0) = 2\cos 0 = 2 \neq 1)$; hence it is not a solution to the initial-value problem. (*b*) $y_2(x)$ satisfies both initial conditions but does not satisfy the differential equation; hence $y_2(x)$ is not a solution. (*c*) $y_3(x)$ satisfies the differential equation and both initial conditions; therefore, it is a solution to the initial-value problem.

1.8. Find the solution to the initial-value problem $y' + y = 0$; $y(3) = 2$, if the general solution to the differential equation is known to be (see Chapter 8) $y(x) = c_1 e^{-x}$, where c_1 is an arbitrary constant.

Since $y(x)$ is a solution of the differential equation for every value of c_1, we seek that value of c_1 which will also satisfy the initial condition. Note that $y(3) = c_1 e^{-3}$. To satisfy the initial condition $y(3) = 2$, it is sufficient to choose c_1 so that $c_1 e^{-3} = 2$, that is, to choose $c_1 = 2e^3$. Substituting this value for c_1 into $y(x)$, we obtain $y(x) = 2e^3 e^{-x} = 2e^{3-x}$ as the solution of the initial-value problem.

1.9. Find a solution to the initial-value problem $y'' + 4y = 0$; $y(0) = 0$, $y'(0) = 1$, if the general solution to the differential equation is known to be (see Chapter 9) $y(x) = c_1 \sin 2x + c_2 \cos 2x$.

Since $y(x)$ is a solution of the differential equation for all values of c_1 and c_2 (see Example 1.4), we seek those values of c_1 and c_2 that will also satisfy the initial conditions. Note that $y(0) = c_1 \sin 0 + c_2 \cos 0 = c_2$. To satisfy the first initial condition, $y(0) = 0$, we choose $c_2 = 0$. Furthermore, $y'(x) = 2c_1 \cos 2x - 2c_2 \sin 2x$; thus,

$y'(0) = 2c_1 \cos\ 0 - 2c_2\ \sin\ 0 = 2c_1$. To satisfy the second initial condition, $y'(0) = 1$, we choose $2c_1 = 1$, or $c_1 = \frac{1}{2}$. Substituting these values of c_1 and c_2 into $y(x)$, we obtain $y(x) = \frac{1}{2}\sin 2x$ as the solution of the initial-value problem.

1.10. Find a solution to the boundary-value problem $y'' + 4y = 0$; $y(\pi/8) = 0$, $y(\pi/6) = 1$, if the general solution to the differential equation is $y(x) = c_1\ \sin\ 2x + c_2\ \cos\ 2x$.

Note that

$$y\left(\frac{\pi}{8}\right) = c_1 \sin\left(\frac{\pi}{4}\right) + c_2 \cos\left(\frac{\pi}{4}\right) = c_1\left(\frac{1}{2}\sqrt{2}\right) + c_2\left(\frac{1}{2}\sqrt{2}\right)$$

To satisfy the condition $y(\pi/8) = 0$, we require

$$c_1\left(\frac{1}{2}\sqrt{2}\right) + c_2\left(\frac{1}{2}\sqrt{2}\right) = 0 \qquad (1)$$

Furthermore,
$$y\left(\frac{\pi}{6}\right) = c_1 \sin\left(\frac{\pi}{3}\right) + c_2 \cos\left(\frac{\pi}{3}\right) = c_1\left(\frac{1}{2}\sqrt{3}\right) + c_2\left(\frac{1}{2}\right)$$

To satisfy the second condition, $y(\pi/6) = 1$, we require

$$\frac{1}{2}\sqrt{3}c_1 + \frac{1}{2}c_2 = 1 \qquad (2)$$

Solving (1) and (2) simultaneously, we find

$$c_1 = -c_2 = \frac{2}{\sqrt{3} - 1}$$

Substituting these values into $y(x)$, we obtain

$$y(x) = \frac{2}{\sqrt{3} - 1}(\sin 2x - \cos 2x)$$

as the solution of the boundary-value problem.

1.11. Find a solution to the boundary-value problem $y'' + 4y = 0$; $y(0) = 1$, $y(\pi/2) = 2$, if the general solution to the differential equation is known to be $y(x) = c_1\ \sin\ 2x + c_2\ \cos\ 2x$.

Since $y(0) = c_1\ \sin\ 0 + c_2\ \cos\ 0 = c_2$, we must choose $c_2 = 1$ to satisfy the condition $y(0) = 1$. Since $y(\pi/2) = c_1 \sin \pi + c_2 \cos \pi = -c_2$, we must choose $c_2 = -2$ to satisfy the second condition, $y(\pi/2) = 2$. Thus, to satisfy both boundary conditions simultaneously, we must require c_2 to equal both 1 and -2, which is impossible. Therefore, there does not exist a solution to this problem.

1.12. Determine c_1 and c_2 so that $y(x) = c_1\ \sin\ 2x + c_2\ \cos\ 2x + 1$ will satisfy the conditions $y(\pi/8) = 0$ and $y'(\pi/8) = \sqrt{2}$.

Note that

$$y\left(\frac{\pi}{8}\right) = c_1 \sin\left(\frac{\pi}{4}\right) + c_2 \cos\left(\frac{\pi}{4}\right) + 1 = c_1\left(\frac{1}{2}\sqrt{2}\right) + c_2\left(\frac{1}{2}\sqrt{2}\right) + 1$$

To satisfy the condition $y(\pi/8) = 0$, we require $c_1(\frac{1}{2}\sqrt{2}) + c_2(\frac{1}{2}\sqrt{2}) + 1 = 0$, or equivalently,

$\frac{\sqrt{2}}{\sqrt{2}}(c_1+c_2)+2=0$

$$c_1 + c_2 = -\sqrt{2} \qquad (1)$$

Since $y'(x) = 2c_1 \cos 2x - 2c_2 \sin 2x$,

$$y'\left(\frac{\pi}{8}\right) = 2c_1 \cos\left(\frac{\pi}{4}\right) - 2c_2 \sin\left(\frac{\pi}{4}\right)$$

$$= 2c_1\left(\frac{1}{2}\sqrt{2}\right) - 2c_2\left(\frac{1}{2}\sqrt{2}\right) = \sqrt{2}c_1 - \sqrt{2}c_2$$

To satisfy the condition $y'(\pi/8) = \sqrt{2}$, we require $\sqrt{2}c_1 - \sqrt{2}c_2 = \sqrt{2}$, or equivalently,

$$c_1 - c_2 = 1 \tag{2}$$

Solving (*1*) and (*2*) simultaneously, we obtain $c_1 = -\frac{1}{2}(\sqrt{2} - 1)$ and $c_2 = -\frac{1}{2}(\sqrt{2} + 1)$.

[redacted] Determine c_1 and c_2 so that $y(x) = c_1 e^{2x} + c_2 e^x + 2 \sin x$ will satisfy the conditions $y(0) = 0$ and $y'(0) = 1$.

Because $\sin 0 = 0$, $y(0) = c_1 + c_2$. To satisfy the condition $y(0) = 0$, we require

$$c_1 + c_2 = 0 \tag{1}$$

From $$y'(x) = 2c_1 e^{2x} + c_2 e^x + 2 \cos x$$

we have $y'(0) = 2c_1 + c_2 + 2$. To satisfy the condition $y'(0) = 1$, we require $2c_1 + c_2 + 2 = 1$, or

$$2c_1 + c_2 = -1 \tag{2}$$

Solving (*1*) and (*2*) simultaneously, we obtain $c_1 = -$ and $c_2 = 1$.

Supplementary Problems

{1.14-1.23} - 1.25 (a, b, e) - 1.27 (a, c, e) - 1.30 (b, c, e) dep.V. - 1.36 - 1.39 - 1.44 - 1.48 1.50?

In Problems 1.14 through 1.23, determine (*a*) the order, (*b*) the unknown function, and (*c*) the independent variable for each of the given differential equations. +degree

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1.14. $(y'')^2 - 3yy' + xy = 0$

1.15. $x^4 y^{(4)} + xy''' = e^x$

1.16. $t^2 \ddot{s} - t\dot{s} = 1 - \sin t$

1.17. $y^{(4)} + xy''' + x^2 y'' - xy' + \sin y = 0$

1.18. $\dfrac{d^n x}{dy^n} = y^2 + 1$

1.19. $\left(\dfrac{d^2 r}{dy^2}\right)^2 + \dfrac{d^2 r}{dy^2} + y\dfrac{dr}{dy} = 0$

1.20. $\left(\dfrac{d^2 y}{dx^2}\right)^{3/2} + y = x$

1.21. $\dfrac{d^7 b}{dp^7} = 3p$

1.22. $\left(\dfrac{db}{dp}\right)^7 = 3p$

1.23. $y^{(6)} + 2y^4 y^{(3)} + 5y^8 = e^x$

1.24. Which of the following functions are solutions of the differential equation $y' - 5y = 0$?

(*a*) $y = 5$, (*b*) $y = 5x$, (*c*) $y = x^5$, (*d*) $y = e^{5x}$, (*e*) $y = 2e^{5x}$, (*f*) $y = 5e^{2x}$

1.25. Which of the following functions are solutions of the differential equation $y' - 3y = 6$?

(*a*) $y = -2$, (*b*) $y = 0$, (*c*) $y = e^{3x} - 2$, (*d*) $y = e^{2x} - 3$, (*e*) $y = 4e^{3x} - 2$

1.26. Which of the following functions are solutions of the differential equation $\dot{y} - 2ty = t$?
(a) $y = 2$, (b) $y = -\frac{1}{2}$, (c) $y = e^{t^2}$, (d) $y = e^{t^2} - \frac{1}{2}$, (e) $y = -7e^{t^2} - \frac{1}{2}$

1.27. Which of the following functions are solutions of the differential equation $dy/dt = y/t$?
(a) $y = 0$, (b) $y = 2$, (c) $y = 2t$, (d) $y = -3t$, (e) $y = t^2$

$\frac{dy}{dt} = y$

1.28. Which of the following functions are solutions of the differential equation

$$\frac{dy}{dx} = \frac{2y^4 + x^4}{xy^3}$$

(a) $y = x$, (b) $y = x^8 - x^4$, (c) $y = \sqrt{x^8 - x^4}$, (d) $y = (x^8 - x^4)^{1/4}$

1.29. Which of the following functions are solutions of the differential equation $y'' - y = 0$?
(a) $y = e^x$, (b) $y = \sin x$, (c) $y = 4e^{-x}$, (d) $y = 0$, (e) $y = \frac{1}{2}x^2 + 1$

1.30. Which of the following functions are solutions of the differential equation $y'' - xy' + y = 0$?
(a) $y = x^2$, (b) $y = x$, (c) $y = 1 - x^2$, (d) $y = 2x^2 - 2$, (e) $y = 0$

1.31. Which of the following functions are solutions of the differential equation $\ddot{x} - 4\dot{x} + 4x = e^t$
(a) $x = e^t$, (b) $x = e^{2t}$, (c) $x = e^{2t} + e^t$, (d) $x = te^{2t} + e^t$, (e) $x = e^{2t} + te^t$

In Problems 1.32 through 1.35, find c so that $x(t) = ce^{2t}$ satisfies the given initial condition.

1.32. $x(0) = 0$ **1.33.** $x(0) = 1$ **1.34.** $x(1) = 1$ **1.35.** $x(2) = -3$

In Problems 1.36 through 1.39, find c so that $y(x) = c(1 - x^2)$ satisfies the given initial condition.

1.36. $y(0) = 1$ **1.37.** $y(1) = 0$ **1.38.** $y(2) = 1$ **1.39.** $y(1) = 2$

In Problems 1.40 through 1.49, find c_1 and c_2 so that $y(x) = c_1 \sin x + c_2 \cos x$ will satisfy the given conditions. Determine whether the given conditions are initial conditions or boundary conditions.

1.40. $y(0) = 1$, $y'(0) = 2$ **1.41.** $y(0) = 2$, $y'(0) = 1$

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1.42. $y\left(\frac{\pi}{2}\right) = 1$, $y'\left(\frac{\pi}{2}\right) = 2$ **1.43.** $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 1$

1.44. $y'(0) = 1$, $y'\left(\frac{\pi}{2}\right) = 1$ **1.45.** $y(0) = 1$, $y'(\pi) = 1$

1.46. $y(0) = 1$, $y(\pi) = 2$ **1.47.** $y(0) = 0$, $y'(0) = 0$

1.48. $y\left(\frac{\pi}{4}\right) = 0$, $y\left(\frac{\pi}{6}\right) = 1$ **1.49.** $y(0) = 0$, $y'\left(\frac{\pi}{2}\right) = 1$

B.C

In Problems 1.50 through 1.54, find values of c_1 and c_2 so that the given functions will satisfy the prescribed initial conditions.

1.50. $y(x) = c_1e^x + c_2e^{-x} + 4\sin x$; $y(0) = 1$, $y'(0) = -1$

1.51. $y(x) = c_1x + c_2 + x^2 - 1$; $y(1) = 1$, $y'(1) = 2$

1.52. $y(x) = c_1e^x + c_2e^{2x} + 3e^{3x}$; $y(0) = 0$, $y'(0) = 0$

1.53. $y(x) = c_1\sin x + c_2\cos x + 1$; $y(\pi) = 0$, $y'(\pi) = 0$

1.54. $y(x) = c_1e^x + c_2xe^x + x^2e^x$; $y(1) = 1$, $y'(1) = -1$